$Nu/D \cong$ const. for $D \geqslant 0.6$ cm. The same parameters are shown in Fig. 4 as a function of the equivalence ratio under quenching conditions.

The case of $D\,Nu =$ const. is equivalent to that of $\lambda/\alpha = \text{const.} \cdot D^2$, and the case of $Nu/D =$ const. is equivalent to that of $\lambda/\alpha =$ const.

It is probable that those relations express the existence of a different behaviour of the reacting mixture under quenching conditions, rather than different heat transfer rules.

Thus, for instance, those relations can be explained by stating that the transition to $Nu/D =$ const. for $D \geqslant 0.6$ cm ($\phi < 0.65$) is accompanied by stabilization of the quantity of heat release, due to an increase in thickness of the wall layers in which the reaction is much slower or does not occur at all.

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MELTING RATES IN TURBULENT RECIRCULATING FLOW SYSTEMS

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NOMENCLATURE

C_p	specific heat of water
d	diameter of the ice rod
h	heat transfer coefficient
H	height of the tank
k	thermal conductivity of water
L	latent heat of melting
r	radial coordinate
r_c	radii of the jet cone at a given level
R	radius of the ice rod
$\mathrm{d}R/\mathrm{d}t$	melting rate of the ice rod
Re	Reynolds number, $U_{\mathrm{loc}} d\rho/\mu$
T	temperature
Tu	turbulence intensity, defined in the text
U_0	centerline velocity
U'	fluctuating velocity
$\bar{U}$	time smoothed velocity
U_{loc}	local velocity
z	axial coordinate.

Greek symbols

μ	viscosity of water
ρ	density of ice.

1. INTRODUCTION

THE PURPOSE of this technical note is to report on experimental measurements concerning the rate at which ice rods melt, when immersed in a pool of water, agitated by an ascending stream of gas bubbles. Problems of this type are of both fundamental and practical interest. From a practical standpoint there are numerous metal processing operations where heat (or mass) transfer between an agitated melt and a solid surface play a central role in determining the overall feasibility or efficiency of the system. The rate at which ferrous or aluminum scrap melts in furnaces, the erosion (corrosion) of refractories in steel processing and the smelting of nickel ores may be cited as examples [1–3].

These problems are of interest from a fundamental standpoint, because while turbulent recirculating flows are reasonably well modelled at present, serious questions remain regarding the appropriate representation of the conditions in the proximity of solid boundaries [4–6].

In a recent paper the present authors described experimental measurements dealing with the velocity profiles and the maps of the turbulent kinetic energy in water, held in a cylindrical container, which was agitated by a gas stream, injected at an axi-symmetrical location at the bottom [7].

The experimental measurements were compared with theoretical predictions based on the numerical solution of the turbulent Navier–Stokes equations, in conjunction with the k model of the turbulent viscosity. In general there was good agreement between the measurements and the predictions.

The investigation to be described here is an extension of this previously described work.

2. EXPERIMENTAL WORK

The apparatus consisted of a cylindrical tank, containing water which was agitated by a gas stream, introduced through the bottom of the container, via a centrally located orifice. Ice rods (frozen onto a steel supporting rod) were immersed into this agitated body of water and the rate of melting was established by photographic means. This cylindrical tank was surrounded by a square tank, also filled with water, in order to eliminate the parallax effects in the optical measurements.

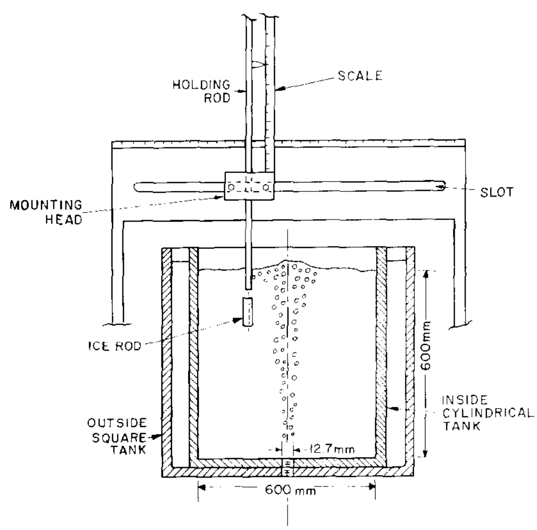

FIG. 1. Schematic sketch of the apparatus.

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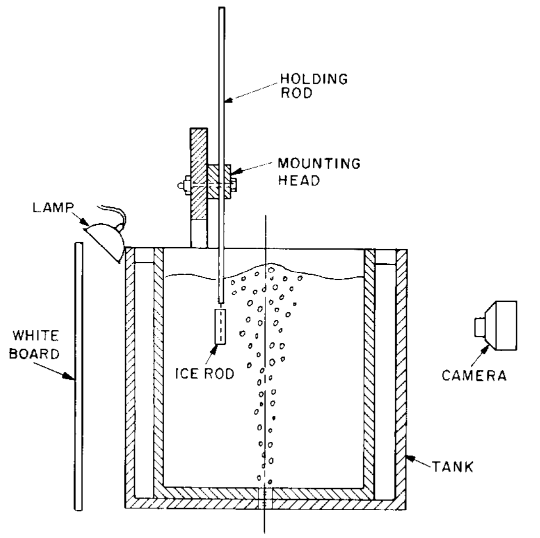

FIG. 2. Sketch of the apparatus, also showing lighting arrangement.

Figure 1 shows a schematic sketch of the apparatus, which was identical to that used in previously reported velocity measurements, using a laser anemometer so that these earlier obtained data on the velocity fields and on the turbulence characteristics of the system could be utilized in the interpretation of the melting rate measurements [7].

The ice rods immersed into the water were supported on a mount which enabled their accurate positioning. Two rod positions were employed; the rods immersed in the bulk of the liquid were held in a vertical position, while the rods immersed in the two-phase region were held in a horizontal position so as to allow the realization of near perpendicular flow to the solid surface. The rods used in the experiments were produced in copper molds, 2.54 cm I.D. and about 8 cm long.

The actual melting rate was determined using a photographic technique. In order to obtain satisfactory contours a special lighting arrangement was made, sketched in Fig. 2. The lighting was provided by a 500 W Sylvania lamp, the illumination being given by light reflected toward the camera by a white board. The photographs were taken in a darkened room. The actual experimental procedure involved the preparation of the ice rods which were then appropriately mounted and immersed into the tank at a predetermined position. Then a photograph was taken of *the rod immersed into quiescent water, which constituted zero time.* Then the air was turned on, giving an airflow rate of 10 Nl min^{-1}, which for the orifice diameter used gave a linear gas velocity of 1.6 m s^{-1} at the inlet. This linear velocity was identical to that used in the previously reported experimental study [7]. Photographs were taken of the dissolving rod at 5–10 s intervals. An Asahi Pentax MX camera was used with a 1.2, 50 mm lens. Kodak Tri X Pan film of 400 ASA rating was used and the shutter speed selected was 1/1000. The temperature in the tank remained essentially constant throughout all the experiments, at a value of about 14.7–15.0°C.

While in principle there may be some concern that a certain time delay had to be experienced before the attainment of steady state, in fact this time delay was quite short, as indicated by the near linearity of the melt line vs time plots, an example of which is shown in Fig. 6.

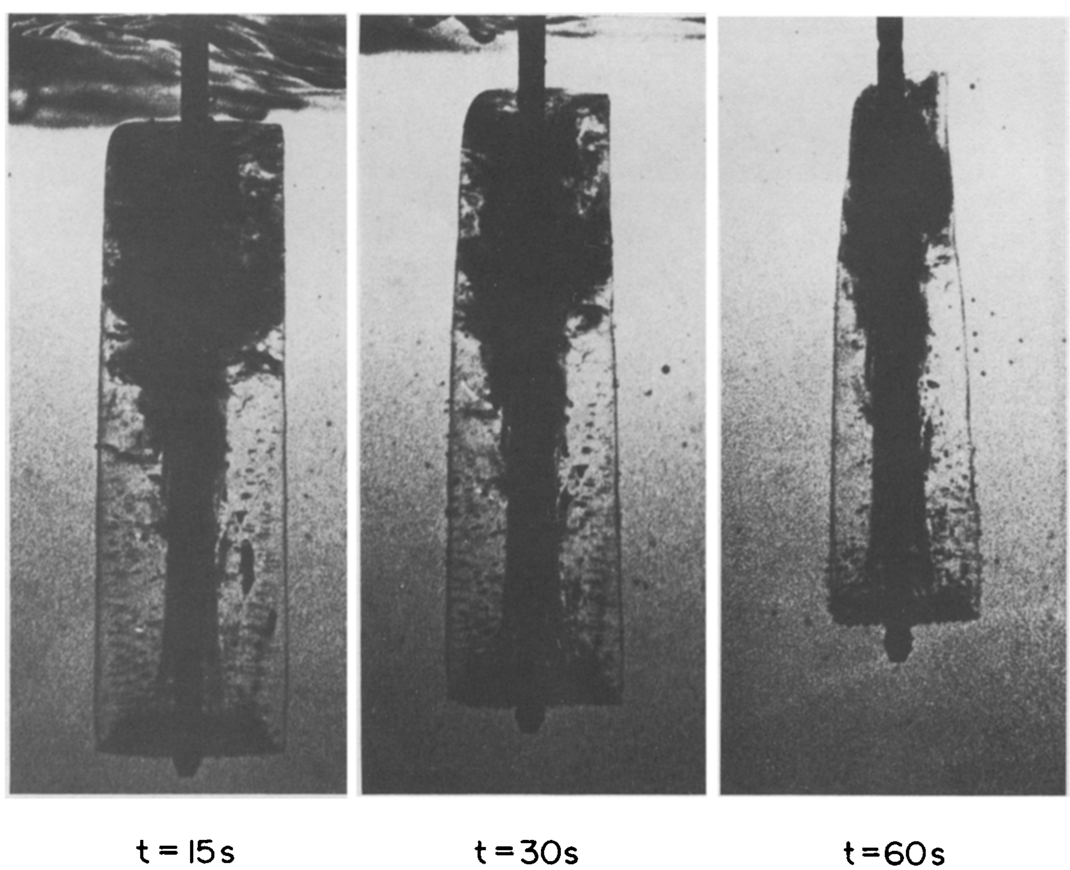

FIG.3. Melting sequence of the ice rod at position $z/H = 0.43$, $r/R = 0.90$, $U_0 = 1.62$ m s^{-1}.

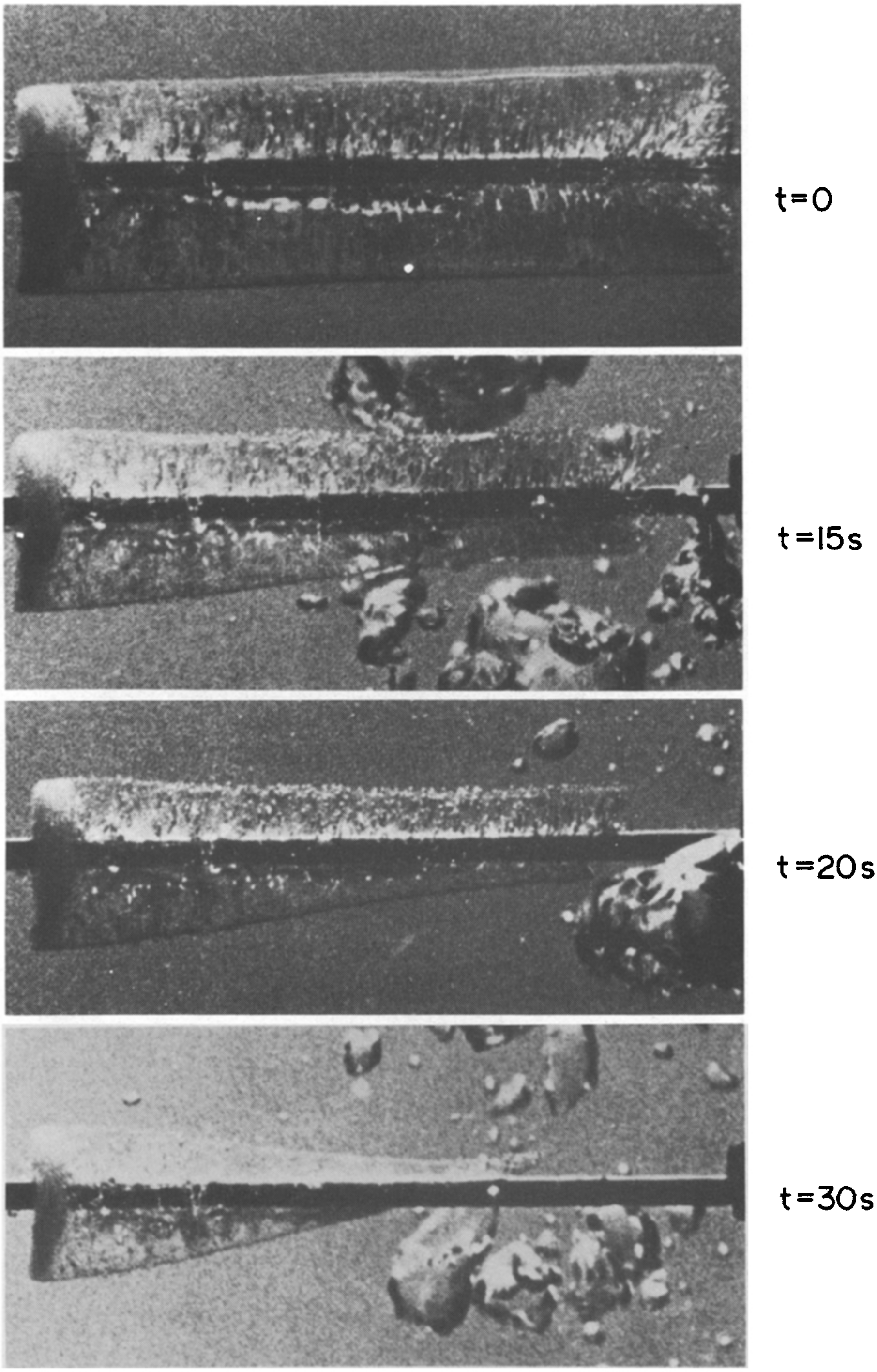

FIG. 4. Melting sequence of the ice rod in the jet cone at $z/H = 0.3$, $U_0 = 1.62$ m s^{-1}.

A large number of experiments were carried out, in course of which the position of the ice rod varied and these will be discussed in the subsequent section.

3. EXPERIMENTAL RESULTS

Figures 3 and 4 show typical photographs of the melting ice rods at various locations, for various values of time. In contrast Fig. 5 shows photographs of the rod, in the absence of airflow. In the absence of airflow the rate of melting was very small; in contrast the turbulent recirculating flow induced by the air stream caused quite appreciable melting rates.

In examining the melting curves it was found that the melting rate was the slowest in the quiescent zone (in the eye of the circulation) significantly enhanced on approaching the free surface and was by far the fastest in the two-phase region.

These findings are consistent with previously reported observations in that the linear velocities and the turbulence levels were the highest in the jet cone, the lowest in the 'eye of the circulation' while the region near the free surface occupied an intermediate position between these extremes.

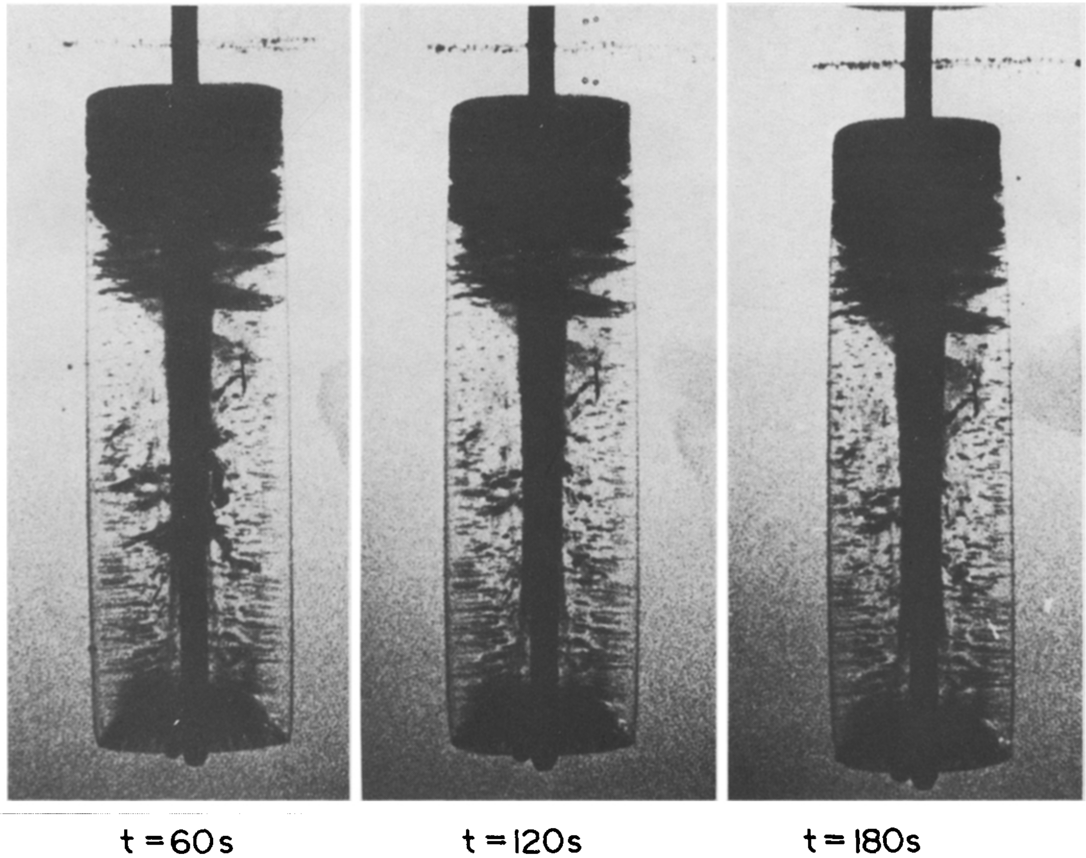

FIG. 5. Melting sequence of the ice rod in stationary fluid at position $z/H = 0.93$, $r/R = 0.9$.

In interpreting these measurements plots were made of the rod diameter against time; a typical example of such a plot is given in Fig. 6. On extrapolating these to time = 0, the initial melting rates were obtained.

Figure 7 shows a typical radial variation of the initial melting rate, which is seen to be the highest at the axis of the two-phase cone, exhibiting quite a marked decrease outside the two-phase region. The variation in the melting rate with the axial position was rather less pronounced as may be seen on comparing Figs. 7 and 8.

In interpreting these measurement rates we have to consider that the melting process is controlled by heat transfer, and may be represented by the following relationship

$$h(T_{\text{lb}} - T_{\text{mp}}) = \rho \Delta L \frac{\mathrm{d}R}{\mathrm{d}t}, \qquad (1)$$

where h is the heat transfer coefficient, T_{lb} is the bulk temperature of the liquid, T_{mp} is the melting temperature of the solid, ρ is the density of the solid, ΔL is the latent heat of melting, and $\mathrm{d}R/\mathrm{d}t$ is the melting rate.

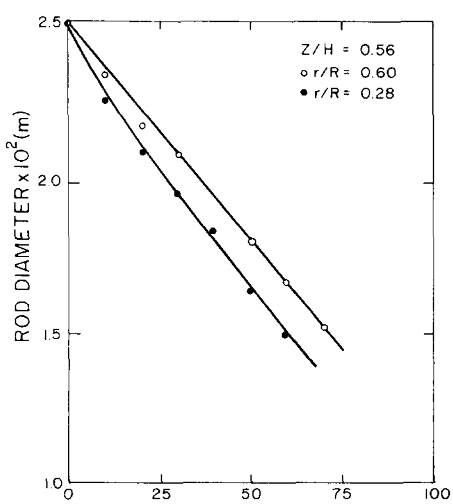

FIG. 6. Ice rod diameter vs time at $z/H = 0.56$.

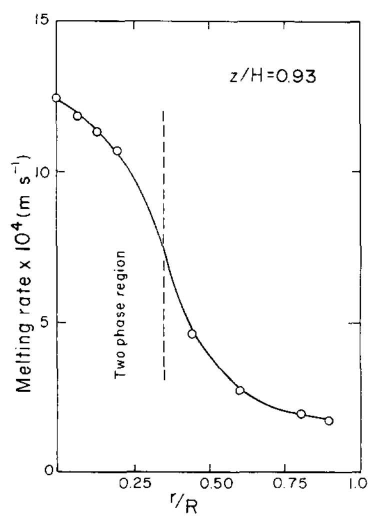

FIG. 7. Radial distribution of the initial melting rate at $z/H = 0.93$.

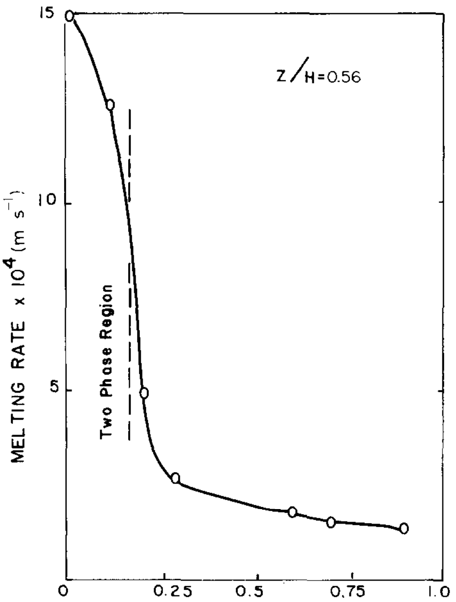

FIG. 8. Radial distribution of the initial melting rate at $z/H = 0.56$.

In many physical situations, for flow past an immersed body in a liquid, the following correlations are used for calculating the heat transfer coefficient

$$Nu = f(Re, Pr), \tag{2}$$

where $Nu = hd/k$ is the Nusselt number, $Re = U\rho d/\mu$ is the Reynolds number, and $Pr = C_p\mu/k$ is the Prandtl number.

On attempting to use correlations of this type the melting rates were significantly underpredicted. This finding is consistent with the suggestion that when there exists significant turbulence in the fluid, this will destabilize the laminar boundary layer and hence cause an enhancement in the transfer rate [8, 9]. Such was likely to be the case in the present situation and also in the vast majority of metal processing operations, involving turbulent recirculating flows in closed systems.

For these conditions the following relationship has been recommended

$$Nu = C_1\ Re^m\ Tu^n\ Pr^{1/3}, \tag{3}$$

where C_1 is a constant and $Tu = \sqrt{U'^2_{\text{loc}}}/\bar{U}_0$ is the turbulence intensity. Here it should be stressed that the definitions used in this work are such that

$$Re = \frac{\bar{U}_{\text{loc}} d\rho}{\mu},$$

i.e. the characteristic velocity in the Reynolds number is the local fluid velocity normal to the rod surface.

In contrast in the definition of Tu, which is identical to that used in ref. [7], $\bar{U}_0$ refers to the velocity in the centerline of the jet.

Both U' and U_{loc} have been mapped out for this system in a previous investigation, which then allows the fitting of appropriate values for C_1, m and n. This is shown in Fig. 9 from which it is seen that $C_1 = 0.338$ and $n = m = 0.8$ seem to give the best fit of the data. Then the correlation proposed for the system takes the following form

$$Nu = 0.388\,(Re\ Tu)^{0.8}\ Pr^{1/3}, \tag{4}$$

$$\text{for } 100 \leqslant Re \leqslant 2000,\ Tu > 0.15$$

$$Pr = 10.$$

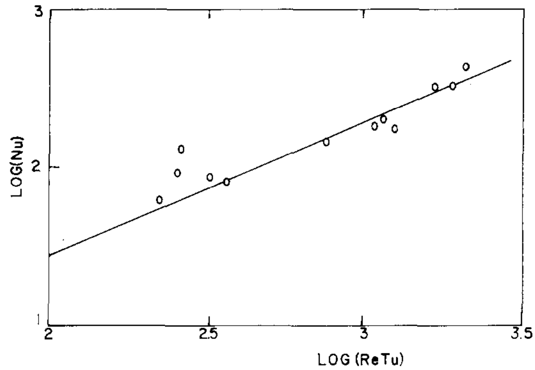

FIG. 9. Relationship between the Nusselt number and the turbulent Reynolds number.

It is noted that the actual numerical values of the Nusselt number that one can deduce from these measurements were found to be very close to the values of the Sherwood number that were obtained from measurements involving the dissolution of graphite rods immersed in a 4 ton argon stirred ladle. The linear fluid velocities and the mean values of the turbulence parameters were comparable in these experiments conducted with molten steel and for the water modelling experiments described in the present study [10].

4. CONCLUSIONS

The important conclusion to be drawn from this work is that when considering heat (or mass) transfer between an agitated melt and an immersed surface, both the linear velocities in the vicinity of the surface and the turbulence levels prevailing in the neighborhood of the melting surface are likely to play an important role. The form of equation (4) suggests that the linear fluid velocity and the turbulence intensity are of equal importance in determining the numerical values of the transfer coefficient. The fact that both Tu and Re are raised to the power 0.8 in equation (4) suggests that the boundary layer surrounding the solid body has in fact turned turbulent, which is quite interesting in view of the relatively low value of the Reynolds number in the system. This finding further stresses the point that the behavior of turbulent recirculating systems may be drastically different from those normally encountered in pipe or channel flow from which most of the conventional correlations are drawn.

As far as mass transfer is concerned, the technique outlined here should provide reasonable predictions for the mass transfer coefficient (utilizing the analogy between heat and mass transfer) because the numerical values of the Schmidt number are likely to be quite similar for most liquids of relatively low viscosity.

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